Efficient Three-Party Shuffling Using Precomputation

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ENCRYPTO CRYPTOGRAPHY AND PRIVACY ENGINEERING

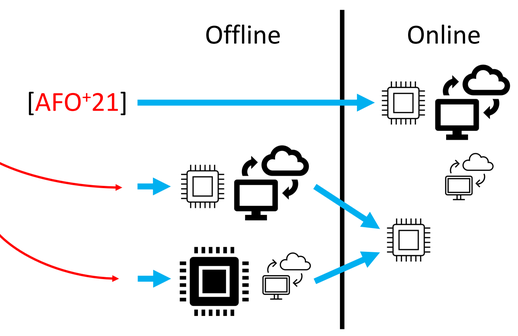
1. Secure Shuffling

- **Goal:** Shuffle private data according to a permutation that is known to no single party
- Applications: contact tracing (Graph Analysis) [AFO+21], metadata-hiding communication [EB22], private database joins [MRR20]
- Mixnets → MPC
- Recent: **RSS based 3PC shuffling [AFO+21],** Clarion: 2PC shuffling with helper [EB22]
- Existing 3PC protocols:

Protocol	Security	Messages per mult./AND	
		Offline	Online
RSS [AFL+16]	semi-honest	—	3
ASTRA [CCPS19]	semi-honest	1	2
SWIFT [PS20]	malicious	3	3

2. Our Contributions

- 3PC shuffling in the **semi-honest** setting
  - **2x online cost improvement** over [AFO+21]
  - Variant I: **Low offline computation**
  - Variant II: **Low offline communication**
- 3PC shuffling in the **malicious** setting
  - Up to **2x online cost improvement** over [AFO+21]
- Insights on **applications** to secure graph analysis



3. Problem Statement

- Input: Matrix $T \in \{0,1\}^{n \times m}$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ - & r_3 & - \\ - & r_4 & - \\ - & r_5 & - \end{pmatrix} \xrightarrow{\pi?} \begin{pmatrix} - & r_4 & - \\ - & r_3 & - \\ 0 & 1 & 1 \\ - & r_5 & - \\ 0 & 1 & 0 \end{pmatrix}$$

- Output: $\pi(T)$ for a random and unknown permutation π on the rows of T

4. General Idea

1. Encode permutation π by matrix M_π

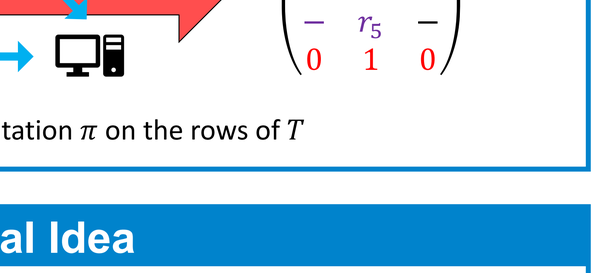
$$\pi = (1,4,5)(2,3) \quad \Rightarrow \quad M_\pi = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2. Secure matrix multiplication

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ - & r_3 & - \\ - & r_4 & - \\ - & r_5 & - \end{pmatrix} = \begin{pmatrix} - & r_4 & - \\ - & r_3 & - \\ 0 & 1 & 1 \\ - & r_5 & - \\ 0 & 1 & 0 \end{pmatrix}$$

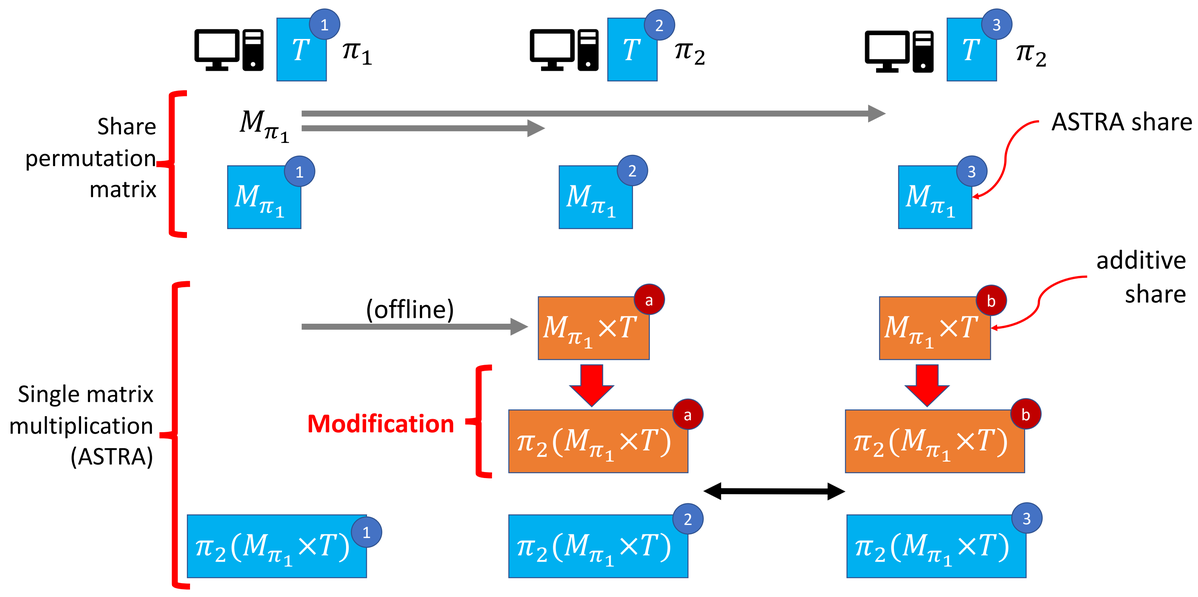
3. Parties generate and secret share multiple random permutation matrices so that no single party knows the product

$$\boldsymbol{M_{\pi_2} \times M_{\pi_1} \times T = (\pi_2 \circ \pi_1)(T)}$$



5. Optimizations

- In ASTRA [CCPS19], the second permutation comes for free



- Share sparse permutation matrix using distributed point functions (**DPF**s) [BGI16]
  - Transfer two keys per matrix row, key length is logarithmic in n

6. Performance

- Summary of permutation costs

Protocol	Communication [bits]		Online rounds
	offline	online	
Semi-honest			
[AFO+21]	—	$4nm$	2
[AFO+21] + offline	$2nm$	**2nm**	2
[EB22]	$2nm$	$3nm$	2
Our variant I	$n^2 + nm$	**2nm**	**1**
Our variant II	$\approx 2n \cdot \log_2(n/\kappa) \cdot \kappa + nm$	**2nm**	**1**
Malicious			
[AFO+21]	—	$> 24\sigma n + 6nm$	$> 9 + 3\log_2 \sigma$
Our variant III	$\approx 4n \cdot \log_2 n \cdot \kappa + 3n^2 + 3nm$	**3nm**	**1**
Our variant IV	$\approx 4n \cdot \log_2 n \cdot \kappa + 6nm$	$6nm$	2

κ/σ: comp./stat. security parameter — Comm. for matrix mult. — DPF key size

- E.g., graph analysis [AFO+21] heavily depends on applying the same permutation to k tables
- Better **amortization** in our protocol

Protocol	Communication [bits]		Online rounds
	offline	online	
[AFO+21]	—	$4knm$	$2k$
Our variant II	$\approx 2n \cdot \log_2(n/\kappa) \cdot \kappa + knm$	**2knm**	**k**

Comm. for matrix mult. — DPF key size — # shuffle runs

- **Open Question**: Efficient inverse of permutation?

References

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DFG Deutsche Forschungsgemeinschaft German Research Foundation

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HESSEN Hessisches Ministerium für Wissenschaft und Kunst